at infinity, the third vertex is the focus of the parabola into which the given conic becomes by projection.

The projection of the second triangle is a circumscribed triangle to the parabola, the circle circumscribing which triangle passing through the focus of the parabola, and proving the theorem.

The reciprocal theorem is: Two triangles are inscribed in a conic; their six sides touch another conic.

150. Proposed by **WILLIAM HOOVER, A.M., Ph.D.,** Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Find the equation to a sphere cutting orthogonally four given spheres.

Solution by the PROPOSER.

Let $x^2+y^2+z^2+2Ax+By+Cz+D=0\ldots.(1)$, be the sphere cutting orthogonally the spheres

$$x^2+y^2+z^2+2a_1x+2b_1y+2c_1z+d_1=0\ldots.(2),$$
$$\Sigma x^2+\Sigma 2a_2x+d_2=0\ldots.(3),$$
$$\Sigma x^2+\Sigma 2a_3x+d_3=0\ldots.(4),$$
$$\Sigma x^2+\Sigma 2a_4x+d_4=0\ldots.(5),$$

Now, two spheres cut each other orthogonally if their radii and the distance between their centers form a right triangle; this requires, for (1) and (2),

$$(A-a_1)^2+(B-b_1)^2+(C-c_1)^2=A^2+B^2+C^2-D+a_1^2+b_1^2+c_1^2-d_1,$$
$$\text{or, } 2Aa_1+2Bb_1+2Cc_1-D-d_1=0\ldots.(6).$$

Similarly for the intersection of (1) and each of (3), (4) and (5),

$$2Aa_2+2Bb_2+2Cc_2-D-d_2=0\ldots.(7),$$
$$2Aa_3+2Bb_3+2Cc_3-D-d_3=0\ldots.(8),$$
$$2Aa_4+2Bb_4+2Cc_4-D-d_4=0\ldots.(9),$$

(6), (7), (8), and (9) give

$$\triangle A=\begin{vmatrix} 2b_1, & 2c_1, & -1, & d_1 \\ 2b_2, & 2c_2, & -1, & d_2 \\ 2b_3, & 2c_3, & -1, & d_3 \\ 2b_4, & 2c_4, & -1, & d_4 \end{vmatrix}\ldots.(10),$$

and like values for B, C, and D, which in (1) gives the required equation.

Similar demonstrations were received from *J. W. YOUNG*, *G. B. M. ZERR*, and *LON C. WALKER*.